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«Application of modular turbo codes in authentication systems»

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Problem statement

The main goal is to use principles of turbo codes and modular codes in order to:

- increase the noise immunity of satellite communication systems;
- provide an authentication procedure for space crafts;
- use the same kind of code for both calculations during the execution of authentication protocol and forwarding information over communication channel.

Solution methods

- Turbo codes

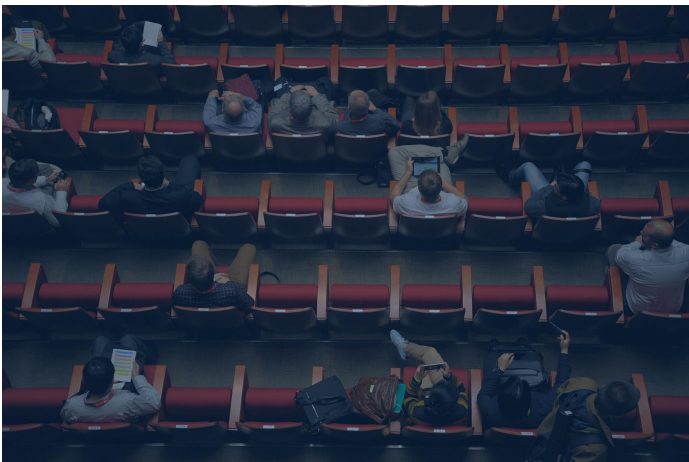
$$\begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} & \dots & \mathbf{a}_{1n} & \mathbf{h}_{11} & \mathbf{h}_{12} & \dots & \mathbf{h}_{1k} \\ \mathbf{a}_{21} & \mathbf{a}_{22} & \dots & \mathbf{a}_{2n} & \mathbf{h}_{21} & \mathbf{h}_{22} & \dots & \mathbf{h}_{2k} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \mathbf{a}_{n1} & \mathbf{a}_{n2} & \dots & \mathbf{a}_{nn} & \mathbf{h}_{n1} & \mathbf{h}_{n2} & \dots & \mathbf{h}_{nk} \\ \mathbf{v}_{11} & \mathbf{v}_{21} & \dots & \mathbf{v}_{n1} & \mathbf{c}_{11} & \mathbf{c}_{12} & \dots & \mathbf{c}_{1k} \\ \mathbf{v}_{12} & \mathbf{v}_{22} & \dots & \mathbf{v}_{n2} & \mathbf{c}_{21} & \mathbf{c}_{22} & \dots & \mathbf{c}_{2k} \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ \mathbf{v}_{1k} & \mathbf{v}_{2k} & \dots & \mathbf{v}_{nk} & \mathbf{c}_{k1} & \mathbf{c}_{k2} & \dots & \mathbf{c}_{kk} \end{pmatrix}$$

- Modular codes

$$\gcd(m_i, m_j) = 1 \quad \forall i \neq j$$

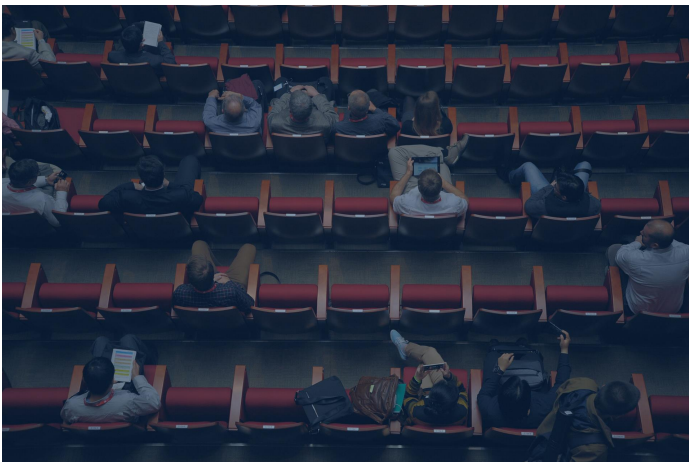
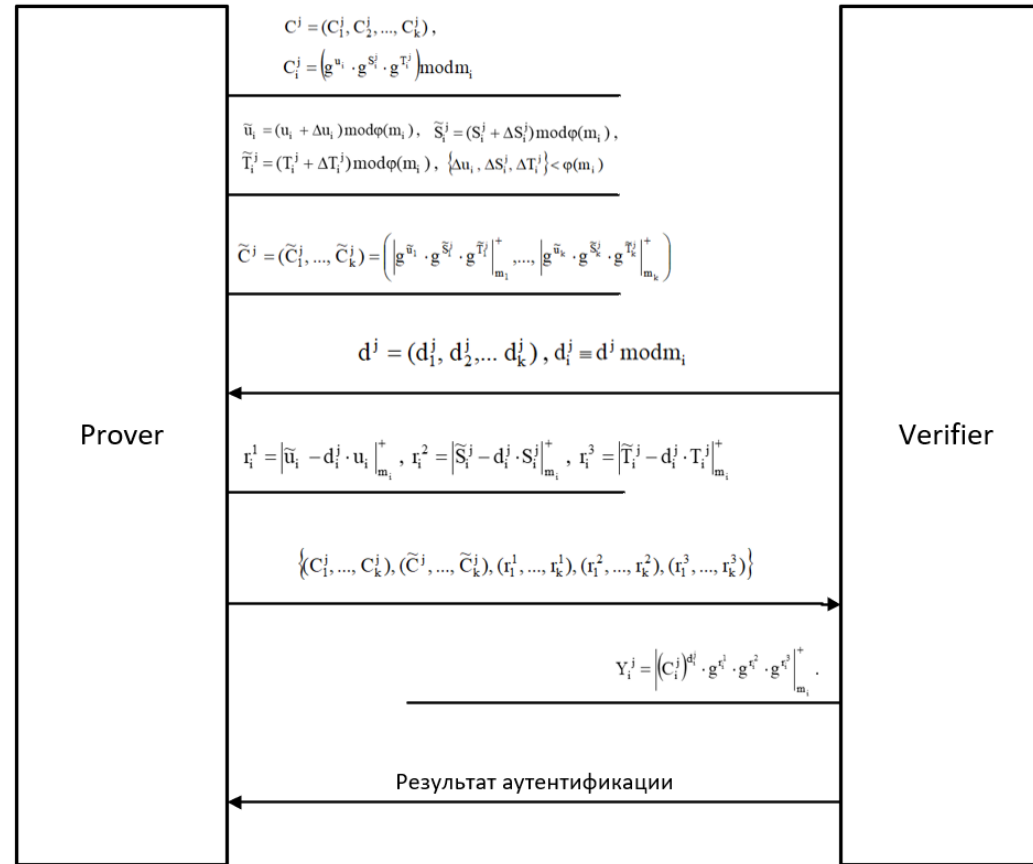
$$\hat{M} = \prod_{i=1}^n m_i, M = \prod_{i=1}^{n+k} m_i = \hat{M} \prod_{i=1}^k m_{n+i};$$

$$Y = (y_1, y_2, \dots, y_n, y_{n+1}, y_{n+2}, \dots, y_{n+k}), y_i = Y \bmod m_i.$$



Solution methods

- Authentication protocol with usage of modular codes



Solution methods

- Structure of the developed modular turbo code

$$\begin{aligned}
 & (C_1^j, C_2^j, C_3^j, C_4^j, C_5^j), \\
 & (\tilde{C}_1^j, \tilde{C}_2^j, \tilde{C}_3^j, \tilde{C}_4^j, \tilde{C}_5^j), \\
 & (r_1^1, r_2^1, r_3^1, r_4^1, r_5^1), \\
 & (r_1^2, r_2^2, r_3^2, r_4^2, r_5^2), \\
 & (r_1^3, r_2^3, r_3^3, r_4^3, r_5^3).
 \end{aligned}
 \Rightarrow A_{RNS} = \begin{cases} (\alpha_{11}, \alpha_{12}, \alpha_{13}, \alpha_{14}, \alpha_{15}) \\ (\alpha_{21}, \alpha_{22}, \alpha_{23}, \alpha_{24}, \alpha_{25}) \\ (\alpha_{31}, \alpha_{32}, \alpha_{33}, \alpha_{34}, \alpha_{35}) \\ (\alpha_{41}, \alpha_{42}, \alpha_{43}, \alpha_{44}, \alpha_{45}) \\ (\alpha_{51}, \alpha_{52}, \alpha_{53}, \alpha_{54}, \alpha_{55}) \end{cases}$$

- Calculation of residues

$$\alpha_{v6}^1 = \left(\sum_{i=1}^5 \alpha_{vi} B_i \bmod \hat{M} \right) \bmod m_6, \alpha_{v7}^1 = \left(\sum_{i=1}^5 \alpha_{vi} B_i \bmod \hat{M} \right) \bmod m_7$$

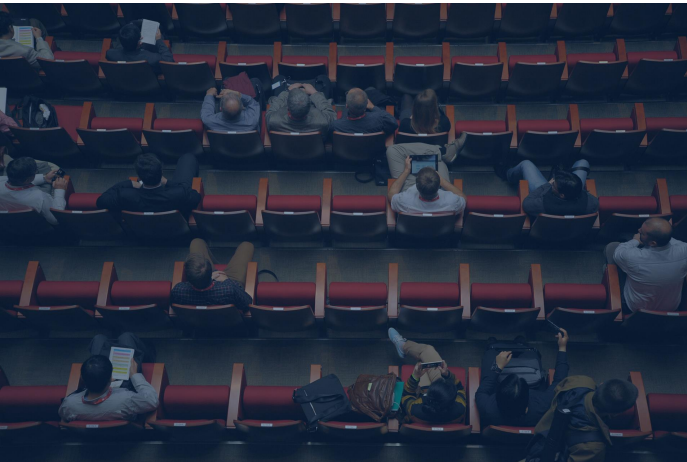
$$\alpha_{1j}^2 = \left(\alpha_{11} B_1 + \alpha_{22} B_2 + \alpha_{33} B_3 + \alpha_{44} B_4 + \alpha_{55} B_5 \right) \bmod \hat{M}_{m_j}^+,$$

$$\alpha_{2j}^2 = \left(\alpha_{21} B_1 + \alpha_{32} B_2 + \alpha_{43} B_3 + \alpha_{44} B_4 + \alpha_{25} B_5 \right) \bmod \hat{M}_{m_j}^+,$$

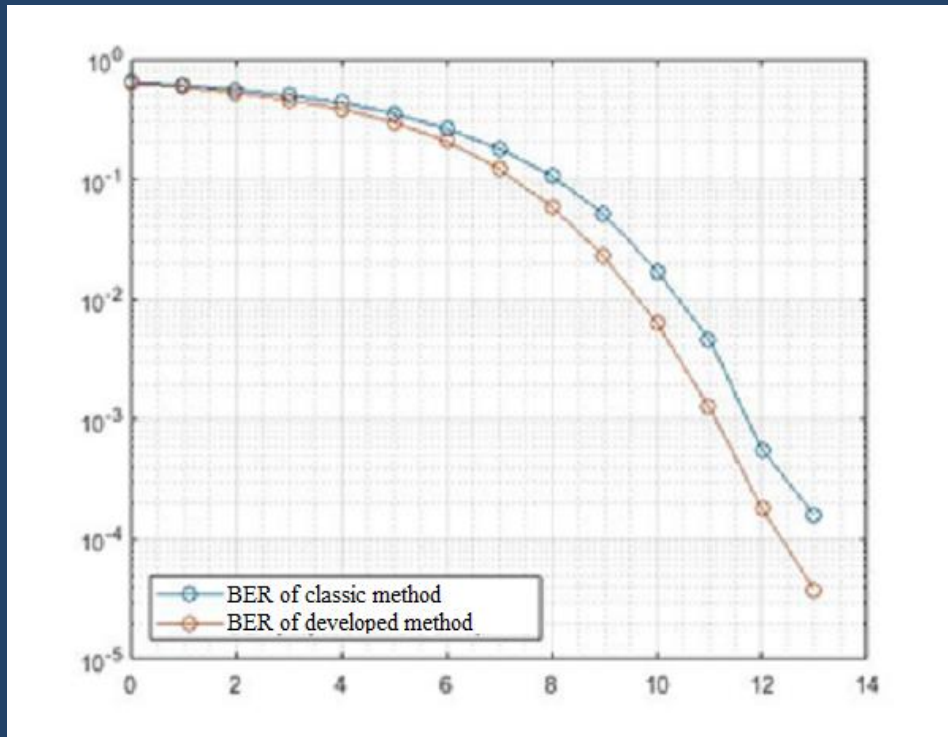
$$\alpha_{3j}^2 = \left(\alpha_{31} B_1 + \alpha_{12} B_2 + \alpha_{53} B_3 + \alpha_{24} B_4 + \alpha_{15} B_5 \right) \bmod \hat{M}_{m_j}^+,$$

$$\alpha_{4j}^2 = \left(\alpha_{41} B_1 + \alpha_{52} B_2 + \alpha_{13} B_3 + \alpha_{14} B_4 + \alpha_{35} B_5 \right) \bmod \hat{M}_{m_j}^+,$$

$$\alpha_{5j}^2 = \left(\alpha_{51} B_1 + \alpha_{42} B_2 + \alpha_{23} B_3 + \alpha_{34} B_4 + \alpha_{55} B_5 \right) \bmod \hat{M}_{m_j}^+.$$



Conclusions



Bit error rates comparison

- Program simulation was created to compare the noise immunity of system for two cases: using not modified authentication protocol and using developed modular turbo code.
- When a signal-to-noise ratio is 13 dB, the probability of a system error using the developed method is $3 \cdot 10^{-5}$ while for the classic RNS it is $8.6 \cdot 10^{-5}$. It means an increase of noise immunity of the system in 2.86 times.

Contacts

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