Modelling and Optimizing Sensor Wireless Network Systems

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Optimality principle and its implementation in the form of a system of functional equations

The mathematical model of the multistep process for making the optimal decision (multistep PMOD) can be written as the following optimization problem:

$$f_N(p_N+1)R(q_1^*,\dots,q_N^*,p_{N+1}) = \min_{(q_1,\dots,q_N)\in D_q}\{\sum_{k=1}^{N} R_k(p_k+1,q_k)\}, \quad (1)$$

where $f_N(p_N+1)-$ minimum value of the generalized performance criterion *N*-step PMOD. Its initial state is characterized by a vector of variables p_{N+1}; $(q_1^*,\dots,q_N^*)-$ the optimal admissible strategy to obtain the value $f_N(p_N+1)R(q_1^*,\dots,q_N^*,p_{N+1})$.

We consider a general approach to solving the optimization problem (1). For this, we construct a family of extremal problems in which the number of steps k is an arbitrary non-negative integer $(0\le k\le N)$. The initial state of p_{k+1} $k-$step confinal subprocess is characterized by a non-negative parameter $p(0\le p\le p_0)$:

$$f_k(p) = \min_{(x_1,\dots,x_k)}\left\{\sum_{i=1}^{k} R_i(x_i)\right\}. \quad (2)$$

in this case, the condition is fulfilled:

$$\sum_{i=1}^{k} x_i \le p;\ x_i\ge 0, i=\overline{1,k}.$$

Computational scheme of the dynamic programming method

We describe the structure of a computational circuit that implements the dynamic programming method using the example of solving a simple distribution problem of the following form :

$$f_N(p_0) = \min_{(x_1,\dots,x_N)}\left\{\sum_{i=1}^{N} R_i(x_i)\right\} \qquad (3)$$

the following condition is considered to be satisfied.

The computational scheme consists of two stages. At the first stage, making assumptions about the values of the variables $p_{k+1}, k = \overline{1,N}$ using the system of recurrence relations (12), the values of the functions $f_k(p_{k+1}), k = \overline{1,N}$ are tabulated. There is also a tabulation of the corresponding optimal controlled parameters $q_k = x_k(p_{k+1})$. The beginning goes from the end of the multistep PMOD (step with the index "1") and further we go to its beginning (the step with the index "N"). Then the sequence of steps involved in making a decision is the reverse of their sequence in time. Assuming k=1, a one-dimensional optimization problem is solved:

$$f_1(p) \min_{x_1=0,1,\dots,\left[\frac{p}{a_1}\right]} R_1(x_1) = \min\left(R_1(0), R_1(1), \dots, R_1(p)\right) \qquad (4)$$

for all integer values $p = 0,1,\dots,p_0$. The result of this procedure is table 1, which consists of three rows that contain the values $p, f_1(p)$ and $x_1(p)$.

Results

Let the following optimization problem be the mathematical model of the performance control n this case, the condition is fulfilled:

$$\max_{(x_1,x_2,x_3)} \{\sum_{i=1}^{3} R_i(x_i)\} \text{ under such condition}, \sum_{i=1}^{3} x_i = 6. \quad (5)$$

$x_i, i = \overline{1,3} -$ non-negative integers. Components x_j are the number of network nodes, the mathematical expectation of the data delivery time from the corresponding node to the required station, and the system uptime. The form of performance indicators for each *k*-th step, which are *s* -functions, is shown in Figure 1.

$$f_1(p_2) = \max_{0 \le x_1 \le p_2}\{R_1(x_1)\} \quad (6)$$

for specific values $p_2 = 0,1,2,\dots,6$.

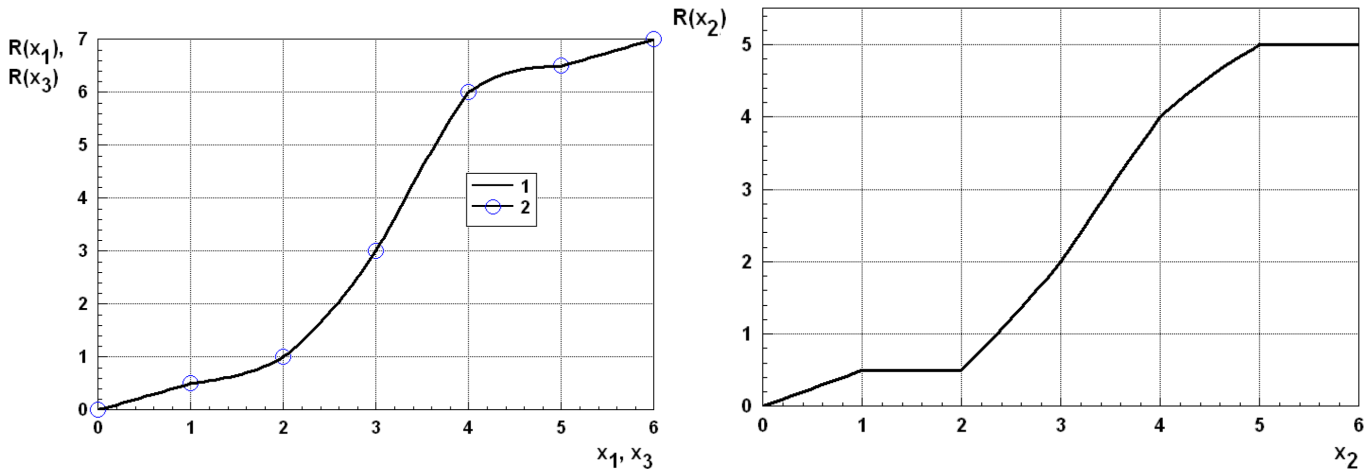

Figure 1. *S* -shaped functions estimating the efficiency $R_k(x_k)$ of decision making at the 1 (curve 1), 2 (curve 2) (a) and 2 (b) steps of a 3-step PMOD

The maximum value of the generalized performance indicator $f_3(6) = 7$ can be obtained using one of the following optimal strategies: $(x_1^*, x_{2,}^* x_3^*) = (6,0,0)$ or $(x_1^*, x_{2,}^* x_3^*) = (5,1,0)$ (Fig/ 2).

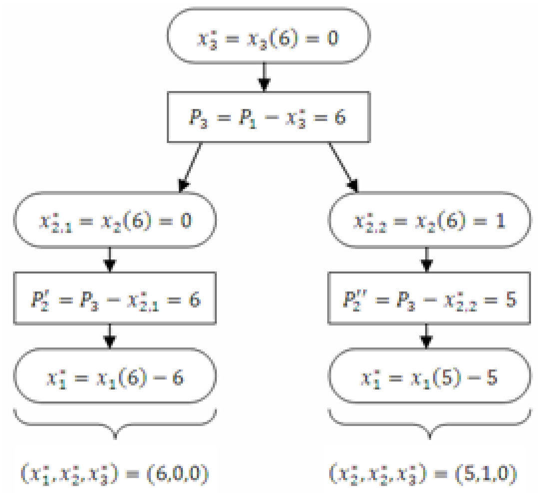

Figure 2. A fragment of the tree of optimal strategies in the case non-uniqueness of the optimal value of the controlled parameter $x_2(p_3)(x_{2,1}^* \neq x_{2,2}^*)$

All possible variants of optimal strategies $(x_1^*, x_{2,}^* x_3^*)$, providing the maximum value $f_3(6) = 7$ during solving the considered problem, are shown in Table 3.

Table 3. Possible variants of optimal strategies

x_3^*	x_2^*	x_1^*	$R_1(x_1^*)$	$R_2(x_3^*)$	$R_3(x_3^*)$	№ of optimal strategy
0	0	6	7	0	0	1
	1	5	6,5	0,5	0	2
1	0	5	6,5	0	0,5	3
	1	4	6	0,5	0,5	4
2	0	4	6	0	1	5
4	0	2	1	0	6	6
	1	1	0,5	0,5	6	7
5	0	1	0,5	0	6,5	8
	1	0	0	0,5	6,5	9
6	0	0	0	0	7	10

Conclusion

In the paper modelling and optimizing of sensor wireless network systems is carried out. The performance characteristics of the wireless sensor network are shown. The general scheme of a multistep process for making an optimal decision is demonstrated. Computational scheme of the dynamic programming method is shown. Some results are obtained that shown optimal strategy during solving the problem.