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Reflection of acoustic waves from a bubble screen in water with hydrate bubbles

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Task definition and main equations

Consider a system of equations describing the propagation of slight disturbance in a bubble screen. The macroscopic equations for maintaining the masses of gas, liquid, and hydrate in linearized form are:

$$\frac{\partial \rho_g}{\partial t} + \rho_{g0}\frac{\partial v}{\partial x} = -J_g,\ \frac{\partial \rho_l}{\partial t} + \rho_{l0}\frac{\partial v}{\partial x} = -J_l,\ \frac{\partial \rho_h}{\partial t} + \rho_{h0}\frac{\partial v}{\partial x} = J_g + J_l \qquad (1)$$

Let us assume there is no bubble splitting. Then their number can be described using the equation

$$\frac{\partial n}{\partial t} + n_0\frac{\partial v}{\partial x} = 0 \qquad (2)$$

The linearized pulse equation is

$$\rho_0\frac{\partial v}{\partial t} = -\frac{\partial p_l}{\partial x} \qquad (3)$$

Here ρ_0 is defined by the formula $\rho_0 = \rho_{l0}^0\alpha_{l0}$.

Changes in liquid temperature around the bubble and gas within the bubble are described by a system of thermal conductivity equations

$$\rho_{l0}^0 c_l\frac{\partial T_l}{\partial t} = r^{-2}\frac{\partial}{\partial r}\left(\lambda_l r^2\frac{\partial T_l}{\partial r}\right),\ (r > a_0), \qquad (4)$$

$$\rho_{g0}^0 c_g\frac{\partial T_g}{\partial t} = r^{-2}\frac{\partial}{\partial r}\left(\lambda_g r^2\frac{\partial T_g}{\partial r}\right) + \frac{\partial p_g}{\partial t},\ (r < a_0) \qquad (5)$$

When the surface tension is neglected, the linearized equation of the bubble pulsation has the form

$$a_0\frac{\partial W_R}{\partial t} + 4\nu_l\frac{W_R}{a_0} = \frac{p_g - p_l}{\rho_{l0}^0} \qquad (6)$$

Here, the dimensionless value R is determined by the formula $R = r/a_0$. Conditions for gas and liquid velocities $r = a_0$ are met at the phase interface

$$\partial a/\partial t = W_l = W_g = W \qquad (7)$$

Heat balance and temperature equality conditions are also met at the phase interface

$$-\lambda_l\left(\frac{\partial T_l}{\partial r}\right)_{r=a_0} + -\lambda_g\left(\frac{\partial T_g}{\partial r}\right)_{r=a_0} = J_h L_h \qquad (8)$$

$$T_g = T_l = T_a, r = a_0 \qquad (9)$$

Symmetry condition is fulfilled in the bubble center, and at the distance from the bubble the liquid temperature is equal to its initial value T_0

$$\partial T_g/\partial r = 0,\ r = 0,\ T_l = T_0, r = \infty \qquad (10)$$

The linearized equation for gas pressure p_g can be written as

$$\frac{\partial p_g}{\partial t} = -\frac{3\gamma p_0}{a_0}\frac{\partial a}{\partial t} + \frac{3(\gamma-1)}{a_0}\lambda_g\left(\frac{\partial T_g}{\partial r}\right)_{r=a} + \frac{3(\gamma-1)}{a_0}c_g T_0 J_g \qquad (11)$$

The rate of hydrate formation is determined by formula:

$$J_g = \frac{\beta(p_h - p_g)}{\sqrt{2\pi R_g T_0}}.\ p_s = p_{s0}\exp\left(\frac{T_a - T_0}{T_*}\right) \qquad (12)$$

Solution of the equations

Formulas for coefficients of reflection (N_1) and passage (M_1) of waves through the first boundary of bubble screen are derived

$$N_1 = \frac{\rho_{z0}^0 C_z - \rho_{l0}^0 C_l}{\rho_{z0}^0 C_z + \rho_{l0}^0 C_l}, M_1 = 1 + N_1 \tag{13}$$

Similarly to formulas (13), coefficients of reflection (N_2) and passage (M_2) of waves through the second boundary of the bubble screen are determined, i.e., when the wave propagating through the bubble screen falls to the bubble screen-liquid boundary.

Expression for the wavenumber

$$K_z^2 = \omega^2 \frac{\alpha_{l0}^2}{C_l^2} + \frac{3\omega^2 \alpha_{l0} \alpha_{g0}}{3\gamma p_0 (\rho_l^0)^{-1} \left(1 + (\gamma - 1)\Pi_g(y_g) - 3/i\omega t_h\right)^{-1} - \omega^2 a_0^2 - 4i\nu_l \omega} \tag{14}$$

Here the parameters Π_g and t_h are determined by formulas

$$\Pi_g(y_g) = 3(y_g \coth y_g - 1) y_g^{-2}, \qquad t_h = \sqrt{2\pi / R_g T_0}\, a_0 / \beta,$$

$$y_g = \sqrt{-i\omega a_0^2 / \chi_g}, \qquad \chi_g = \lambda_g / \rho_{g0}^0 c_{pg}$$

Calculation results

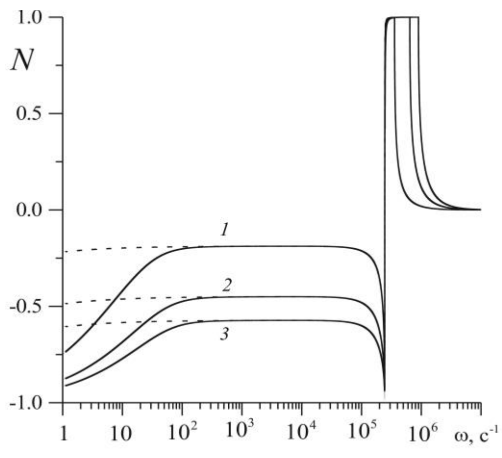

Figure 1. Dependence of reflection coefficient on circular frequency at different values of gas volume. Solid lines are obtained taking into account the process of hydrate formation, while dotted lines - without the process of hydrate formation.

Calculation results

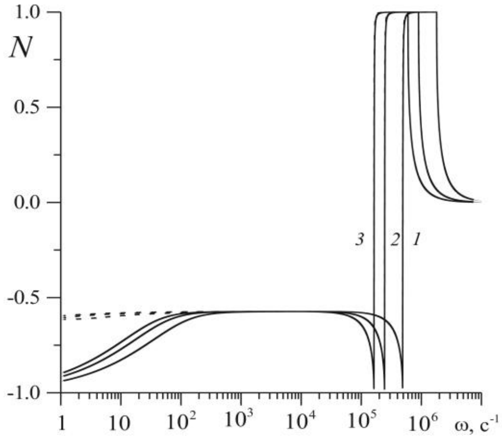

Figure2. Dependences of coefficient of reflection on circular frequency at different values of radius of a bubble. Solid lines are obtained taking into account the process of hydrate formation, while dotted lines - without the process of hydrate formation.