Automated Verification of Expression Transformation Chains Based on Computational Experiments

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Problem of Symbolic Expressions Comparison

- The solution of mathematical and logical problems is the imperative element of the educational process in the technical sciences. To solidify the assimilated skills, learner need to solve a large number of educational tasks, that leads to the need of checking of a large number of solutions. In lots of cases the solution of an educational task is the chain of expression transformations.
- Many systems allow to compare the learner’s answer with the correct one, defined by the teacher. But in some cases, the teacher needs to set some additional information, besides the answer expression. Such additional information cannot be set for each possible pair of expressions in each possible solution chain of the task.
- We propose a method for verifying task solutions, based on the computational experiments between each pair of steps in solution chain. The proposed approach was tested and showed good results. Namely, in the experiments the efficiency of the teacher's work when checking solutions increased more than 4 times.

Proposed Approach of Expressions Comparison

1. The system chooses sets of values of the symbolic variables in the expressions by the uniform distribution.
2. For each set of values of the variables system checks equality of the expressions in real numbers.
3. If the system finds a counterexample, in which expressions are different, then expressions are considered as unequal. Otherwise and if expressions are defined in all sets of values, expressions are considered as equal.
4. For each set of values of variables in which expressions are undefined, the system checks equality of the expressions in complex numbers fixing one holomorphic branch between 0 and 2π.
5. The system estimates the probability for expressions being equal and compares it with a ratio of the number of points where expressions are equal to the number of points where expressions differ. If the probability is bigger, expressions are considered as equal, otherwise unequal.

Probability of Expressions Being Equal

$$\ln\left(\prod_{j=1}^{n} z_j\right) = \sum_{j=1}^{n} \ln\left(z_j\right)$$

$$\ln\left(\prod_{j=1}^{n} z_j\right) = \ln\left(e^{i\sum_{j=1}^{n}\varphi_j}\prod_{j=1}^{n} r_j\right) =$$

$$= \ln\left(\prod_{j=1}^{n} r_j\right) + i\left(\sum_{j=1}^{n}\varphi_j - 2\pi k\right) =$$

$$= \sum_{j=1}^{n} \ln\left(r_j\right) + i\left(\sum_{j=1}^{n}\varphi_j - 2\pi k\right)$$

where $k\epsilon Z$ is selected to satisfy $0 \le \sum_{j=1}^{n}\varphi_j - 2\pi k < 2\pi$.

$$\sum_{j=1}^{n} \ln\left(z_j\right) = \sum_{j=1}^{n}\left(\ln(r_j) + i\varphi_j\right) = \sum_{j=1}^{n} \ln(r_j) + i\sum_{j=1}^{n}\varphi_j$$

Equal if and only if

$$0 \le \sum_{j=1}^{n}\varphi_j < 2\pi.$$

the probability is $\frac{1}{n!}$

Thanks for the Attention!

- The proposed extension for the computational experiments method significantly expands the prospects for verifying solutions of typical educational tasks. The results of using this method substantially depend on the degree of ease with which the learner perceives the recording of problem solutions, which should also be understood by the checking system. The proposed extension significantly increases the number of ways of the recording solutions, allows to omit unnecessary details.
- We keep working in such areas, our future prospects of the research are to support new function and operation types in new subject areas and to continue to improve the notation for the recording of solutions.