Two problems of movement of multi-link wheeled vehicles

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Mathematical formulation of the problem

A system consisting of n trolleys moves along a plane. One wheel pair is fixed on the trolleys so that the center of mass of the link coincides with the geometric center of the wheel pair and lies on the axis of symmetry of the trolley. Each trolley is rigidly fixed with a metal frame of length l. The frame attaches the trolley to the previous trolley in its geometric center and rotates freely around the attachment point.

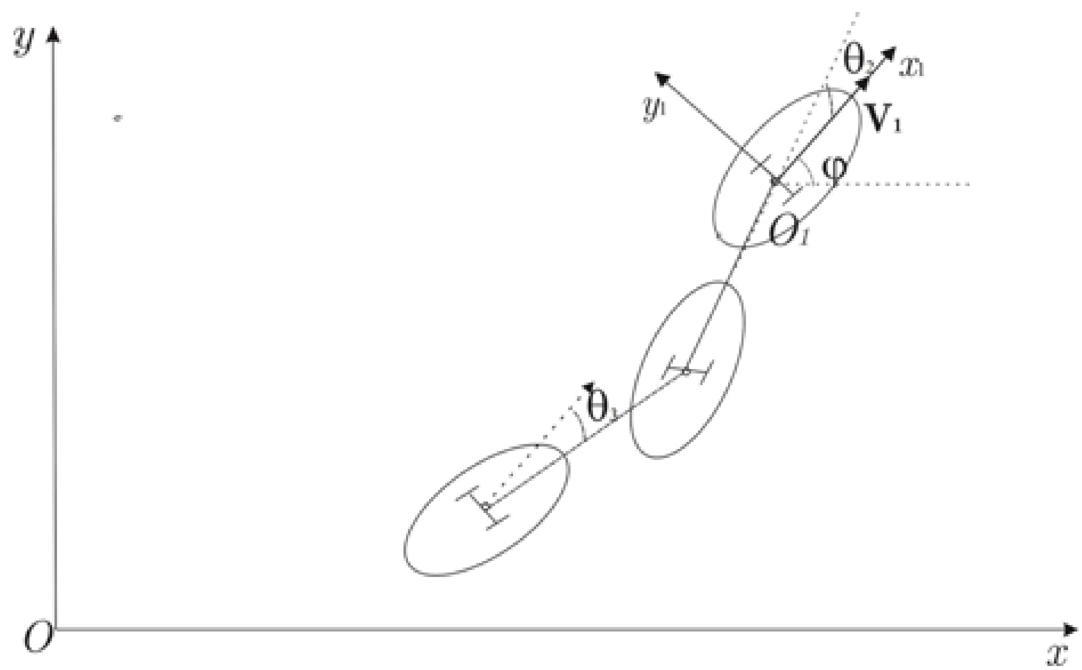


Figure 1. Construction of a multi-link wheeled vehicle.

The kinetic energy will take the form

$$T=\frac{1}{2}I_1\omega_1{}^2+\frac{1}{2}\sum_{i=2}^{n}J_i\left(\omega_1+\dot\theta_i\right)^2+\frac{Mn}{2}\left(u^2+v^2\right)+Ml\sum_{i=2}^{n}\left(n+1-i\right)\left(\omega_1+\dot\theta_i\right)\left(u\sin\theta_i-v\cos\theta_i\right)+$$
$$+Ml^2\sum_{m=2}^{n-1}\sum_{i=m+1}^{n}\left(n+1-i\right)\left(\omega_1+\dot\theta_m\right)\left(\omega_1+\dot\theta_i\right)\cos\left(\theta_i-\theta_m\right), \quad (1)$$

where $J_i=I_i+Ml^2(n+1-i)$.

Nonholonomic constraints are imposed on the wheels at each of the links; these constraints mean that the velocity vector of the center of mass of the trolley is always co-directional with the axis of the trolley:

$$f_1=v=0,$$
$$f_i=u\sin\theta_i+l\sum_{k=2}^{i-1}(\omega_1+\dot\theta_k)\cos\left(\theta_i-\theta_k\right)+l(\omega_1+\dot\theta_i)=0. \quad (2)$$

We believe that $\sum_{k=2}^{1}g_k=0$.

Equations of motion with indefinite multipliers will take the form

$$\frac{d}{dt}\left(\frac{\partial T}{\partial u}\right)-\omega_1\frac{\partial T}{\partial v}=\sum_{i=1}^{n}\lambda_i\frac{\partial f_i}{\partial u},$$
$$\frac{d}{dt}\left(\frac{\partial T}{\partial v}\right)+\omega_1\frac{\partial T}{\partial u}=\sum_{i=1}^{n}\lambda_i\frac{\partial f_i}{\partial v},$$
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \omega_1}\right)+u\frac{\partial T}{\partial v}-v\frac{\partial T}{\partial u}=\sum_{i=1}^{n}\lambda_i\frac{\partial f_i}{\partial \omega_1}, \quad (3)$$
$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot\theta_i}\right)-\frac{\partial T}{\partial \theta_i}=\sum_{i=1}^{n}\lambda_i\frac{\partial f_i}{\partial \dot\theta_i}.$$

A controlled wheeled vehicle

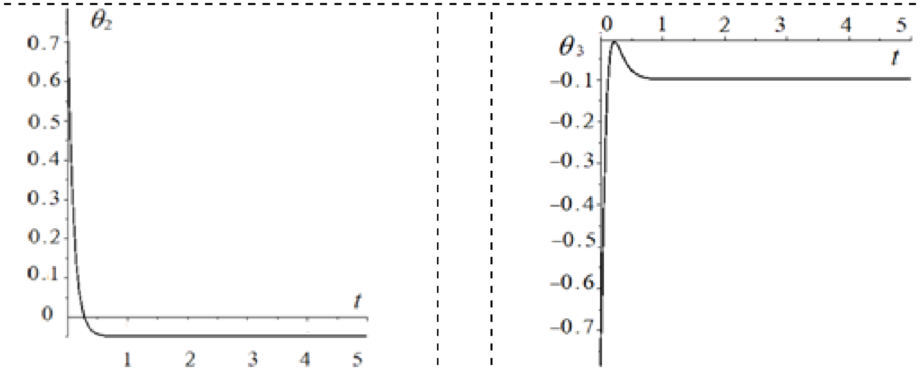
Example. Let the linear velocity and angular velocity of the center of mass of the first link of a three-link wheeled vehicle be given:

$$u = u_0, \ \ \omega_1 = \Omega, \quad \varphi = \Omega t \ .$$

Then the angles θ_2, θ_3 are determined from the differential equations:

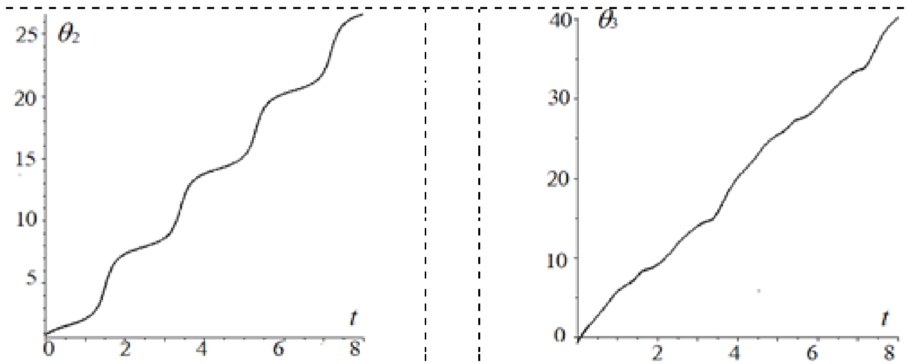
$$\begin{aligned} \dot{\theta}_2 &= -\frac{u_0 \sin\theta_2}{l} - \Omega, \\ \dot{\theta}_3 &= -\frac{u_0 \cos\theta_2 \sin\left(\theta_3 - \theta_2\right)}{l} - \Omega. \end{aligned} \tag{4}$$

Equations (4) can be solved both numerically and analytically. Figure 2 and figure 3 shows graphs of the desired functions θ_2, θ_3 for different various input parameters.



(a) **(b)**

Figure 2. Graphs of functions θ_2, θ_3 for $l = 1, \Omega = 0.5, u_0 = 10$.



(a) **(b)**

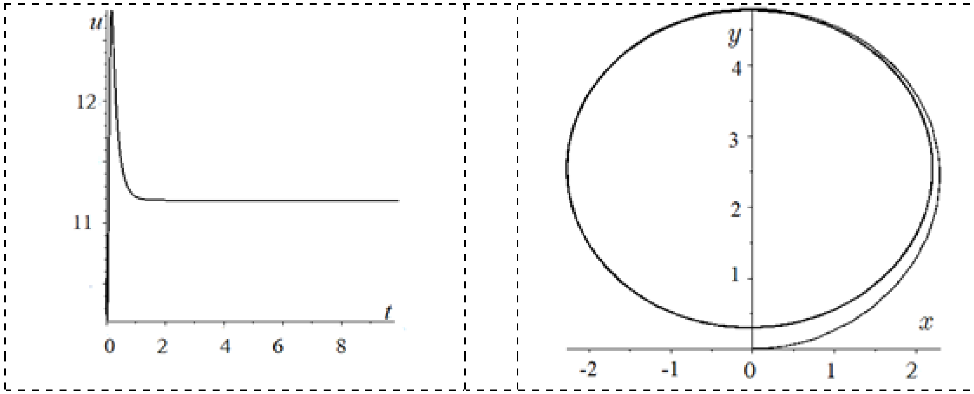
Figure 3. Graphs of functions θ_2, θ_3 for $l = 6, \Omega = -2, u_0 = 10$.

An uncontrolled wheeled vehicle

We will assume that the law of motion of the first cart is unknown. In this case, after the joint solution of equations (3) and derivatives of the time coupling equations (2) and reduction to the level of the first integrals, we obtain the reduced system for three trolleys.

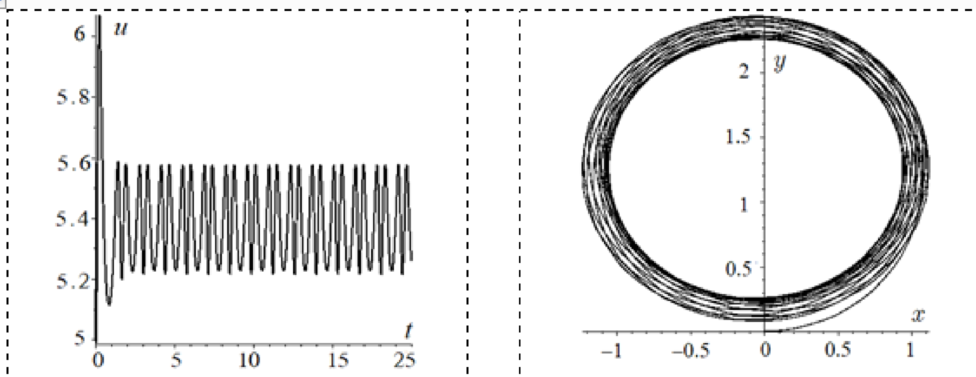
$$\begin{gathered}
\dot{u} = \frac{1}{R(\theta_2,\theta_3)}\Big[\sin\theta_2\cos\theta_2\left((J_2-4)-(J_3-2)\sin^2\alpha\right)u\omega_1+\Big((J_2-4)\sin^2\theta_2\cos\theta_2+ \\
+(J_3-2)\cos\theta_2\sin\alpha\left(-\sin\theta_2\cos\theta_2\cos\alpha-\sin^2\theta_2\sin\alpha+\cos^2\theta_2\cos\alpha\sin\alpha\right)\Big)u^2\Big], \\
\dot{\omega}_1 = 0, \\
\dot{\theta}_2 = -u\sin\theta_2-\omega_1, \\
\dot{\theta}_3 = -u\cos\theta_2\sin\alpha-\omega_1,
\end{gathered} \tag{5}$$

where $M = l = 1$, $\alpha = \theta_3 - \theta_2$, $R(\theta_2,\theta_3) = (J_2-4)\sin^2\theta_2 + (J_3-2)\cos^2\theta_2\sin^2\alpha + 3$.



(a) (b)

Figure 4. Graph of function u and trajectory of the center of mass of the first cart for $I_1 = 4, J_2 = 5, J_3 = 6, F = 600, \omega_0 = 5, \theta_2(0) = \frac{\pi}{4}, \theta_3(0) = \frac{\pi}{2}$.



(a) (b)

Figure 5. Graph of function u and trajectory of the center of mass of the first cart for $I_1 = 4, J_2 = 5, J_3 = 6, F = 220, \omega_0 = 5, \theta_2(0) = \frac{\pi}{4}, \theta_3(0) = \frac{\pi}{2}$.