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Advances in Materials Sciences - 2021

Mapping of Beta Distribution for the Study of Dispersed Material

Polosin Vitaly Germanovich

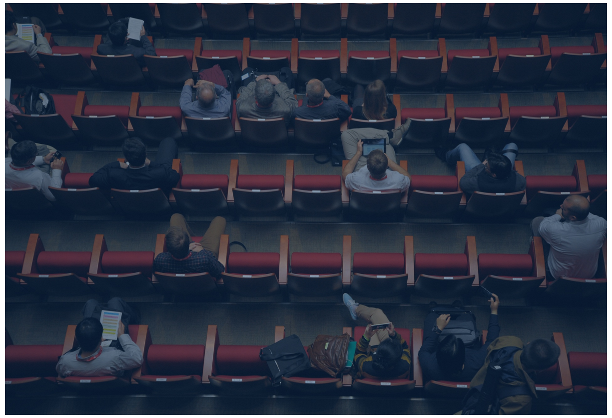
Problem statement

- In the study of polydisperse materials, most of the experimental particle size distributions were obtained on bounded intervas
- The beta distribution is used to approximate results due to the fact that this distribution contains many shape for displaying realizations on a bounded interval.
- The possibilities of using known shape measures for mapping beta distribution is discusses.

- The purpose is concluded to develop methods for identifying the shape of the particle size distributions models for the polydisperse materials by analyzing experimental data in the space of probable and informational sing of the shape of the beta distribution model.

Solution method

Beta distribution models is a shape manifold that are given by two parameters



- Beta distribution models of the particle size distributions

$$f(y;u,v,a,b)=\frac{(y-a)^{u-1}(b-y)^{v}}{(b-a)^{u+v-1}\mathrm{B}(u,v)}.$$

- This is a linear transformation for changing the location and scale of the distribution of a random variable *Y*, that takes nonzero values only on bounded intervas [a, b].

$$x=\frac{y-a}{b-a}.$$

- The beta distribution for determining shape parameters

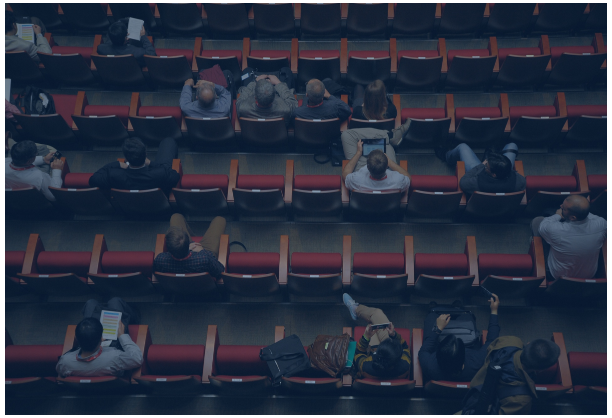
$$f(x)=\frac{x^{u-1}(1-x)^{v-1}}{\mathrm{B}(u,v)},\quad 0<x<1$$

- Where B(*u*,*v*) is a special beta function of two variables that is given by the Euler integral of the first kind

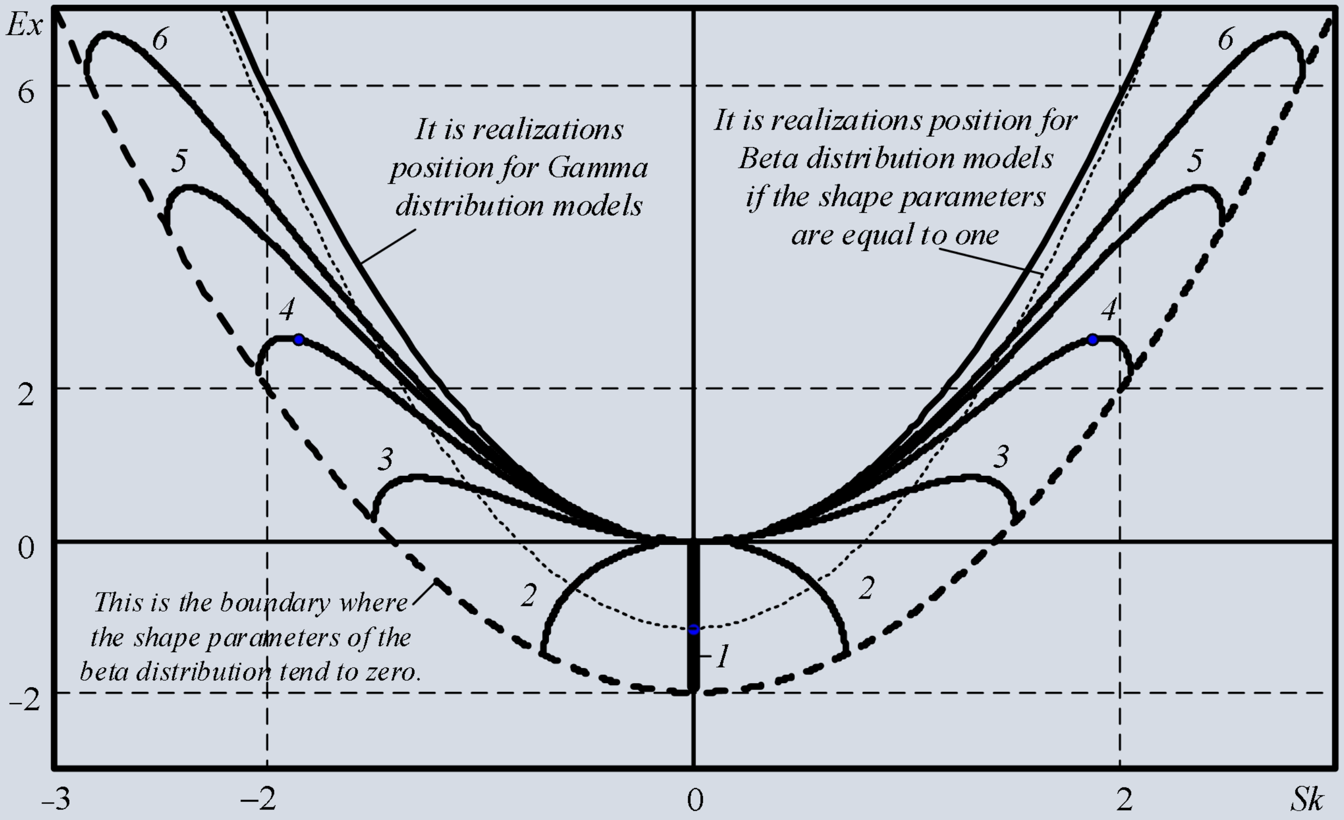
$$\mathrm{B}(u,v)=\int_0^1 t^{u-1}(1-t)^{v-1}\,dt.$$

Solution method

Lack of method. Realizations with different ratios n overlap if the shape parameters are greater than 1



- The space of probabilistic shape measures of the Excess-kurtosis and Skewness for beta distribution implementations



where the numbers of 1, 2, 3, 4, 5 and 6 show the possible trajectories position of beta distributionsif the *n* ratio of shape parameters is equal to 1, 2, 4, 6, 8 and 10, respectively.

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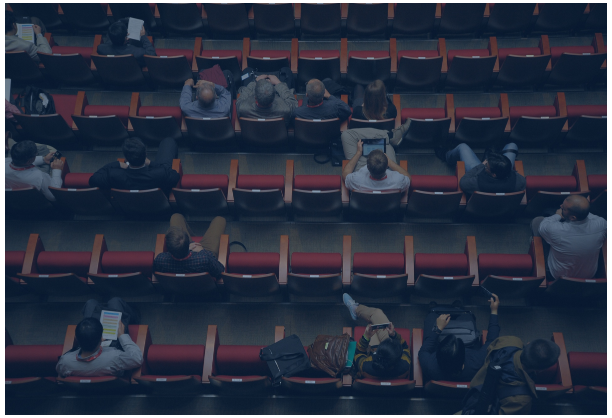
Solution method

It were obtained the new shape measures for non-shifted and shifted models.



- Information entropy as an independent measure of the distribution uncertainty that contain information about the distribution shape

$$H(u,v) = \ln B(u,v) - (u-1)\psi(u) - (v-1)\psi(v) + (u+v-2)\psi(u+v).$$

The entropy coefficient for the family of non-shifted beta distributions of the first type

$$K_{HnB}(u,v) = \sqrt{\frac{(u+v)(u+v+1)}{u(u+1)}}\,\left[\mathrm{B}(u,v)\right]\exp\left[(u+v-2)\psi(u+v)-(u-1)\psi(u)-(v-1)\psi(v)\right]$$

The entropy coefficient for the family of beta distributions of the first type that based on the central moment

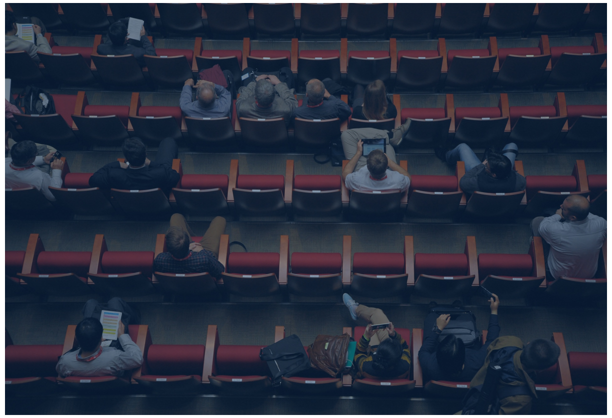
$$K_{HsB}(u,v) = \frac{(u+v)}{2}\sqrt{\frac{u+v+1}{uv}}\left[B(u,v)\right]\exp\left[(u+v-2)\psi(u+v)-(u-1)\psi(u)-(\beta-1)\psi(\beta)\right]$$

The entropy coefficient for the symmetric model of the beta distributions

$$K_{HsB}(u) = \sqrt{(2u+1)}\left[B(u,u)\right]\exp\left[2(u-1)\left(\psi(2u)-\psi(u)\right)\right]$$

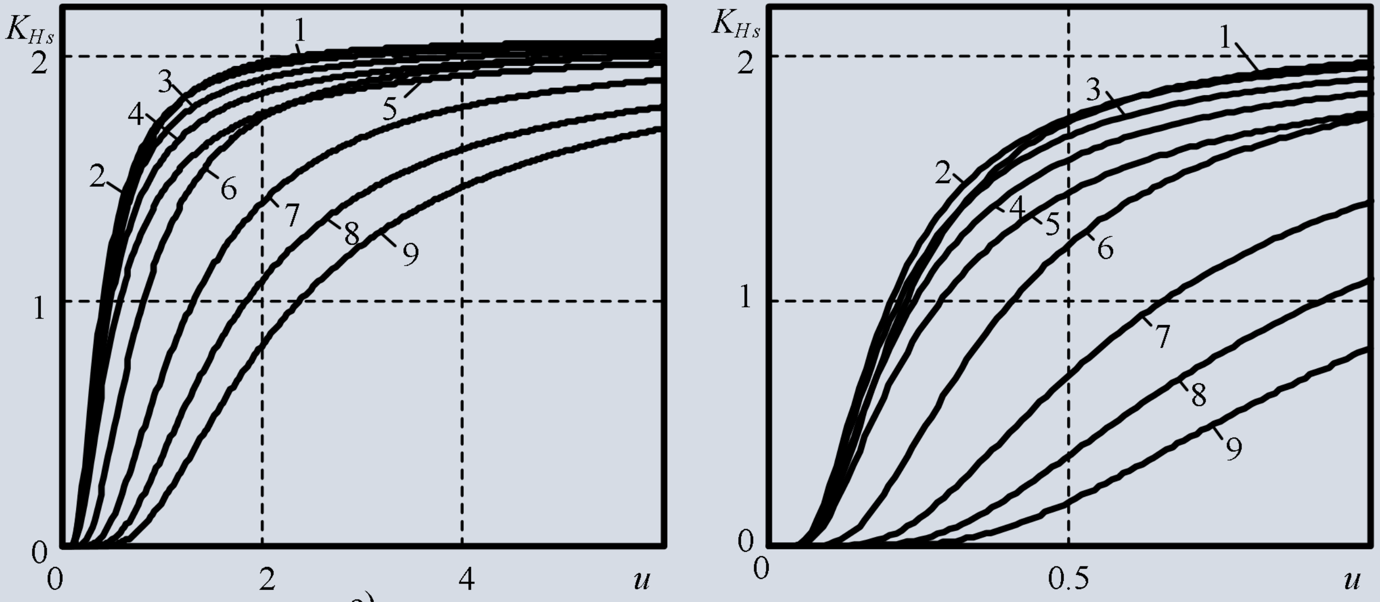
Solution method

This is disadvantage if there is a low variability for the entropy coefficient when the shape parameters are greater than one.



- The entropy coefficient for beta distributions that based on the central moment

$$K_{Hs\mathrm{B}}(u,v)=\frac{(u+v)}{2}\sqrt{\frac{u+v+1}{uv}}\left[B(u,v)\right]\exp\left[(u+v-2)\psi(u+v)-(u-1)\psi(u)-(\beta-1)\psi(\beta)\right]$$



a) b)

If the v second shape parameter is larger than the u first parameter by 2, 3, 5 and 15 times, there are the graphs with numbers of 2, 3, 4 and 5 respectively. In the same place, numbers of 6, 7, 8 and 9 is graphs of the change in the entropy coefficient from the v second shape parameter provided that the u first shape parameter is also 2, 3, 5 and 15 times greater than the v second parameter.

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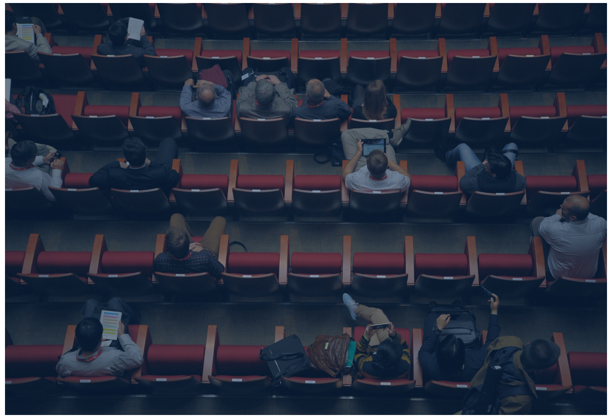
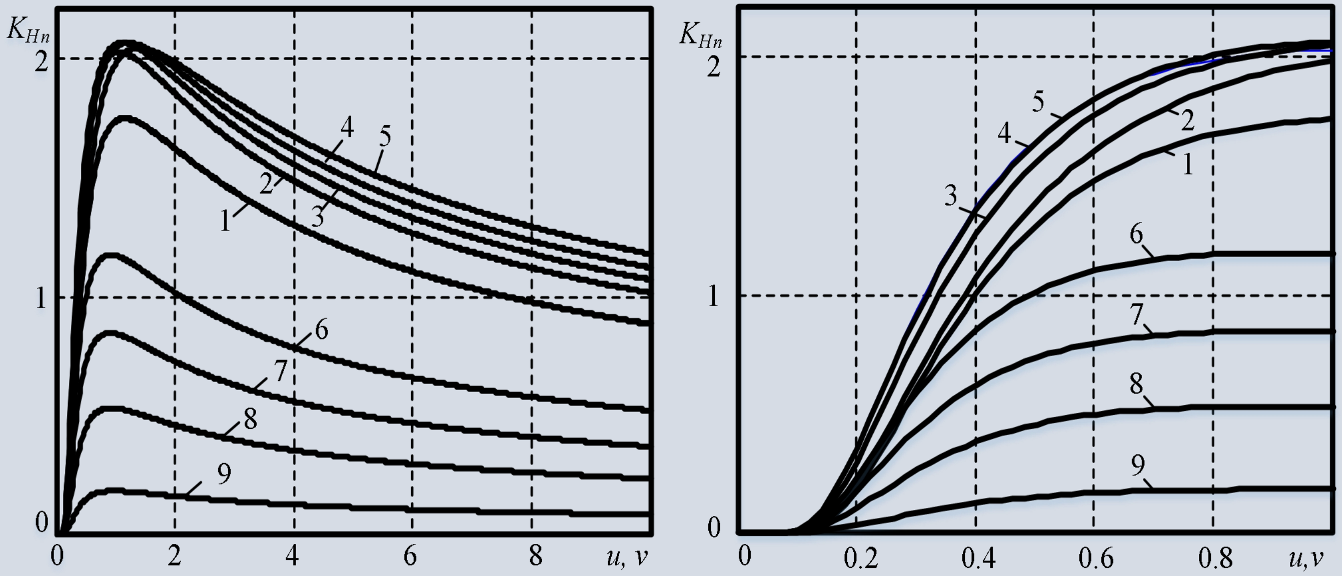
Solution method

This is an advantage if the entropy coefficient changes for any shape parameters



- The entropy coefficient for the non-shifted beta distributions of the first type

$$K_{Hn\mathrm{B}}(u,v)=\sqrt{\frac{(u+v)(u+v+1)}{u(u+1)}}\,\left[\mathrm{B}(u,v)\right]\exp\left[(u+v-2)\psi(u+v)-(u-1)\psi(u)-(v-1)\psi(v)\right]$$

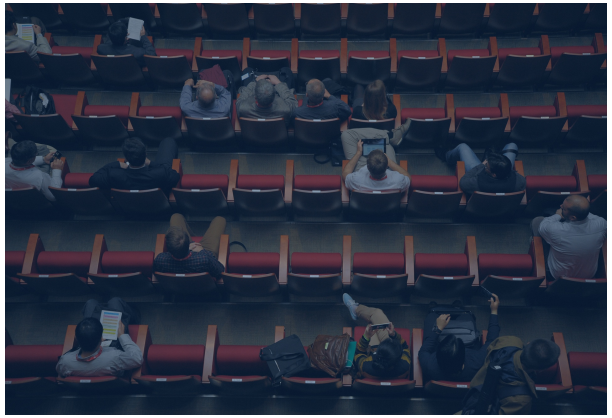


a) b)

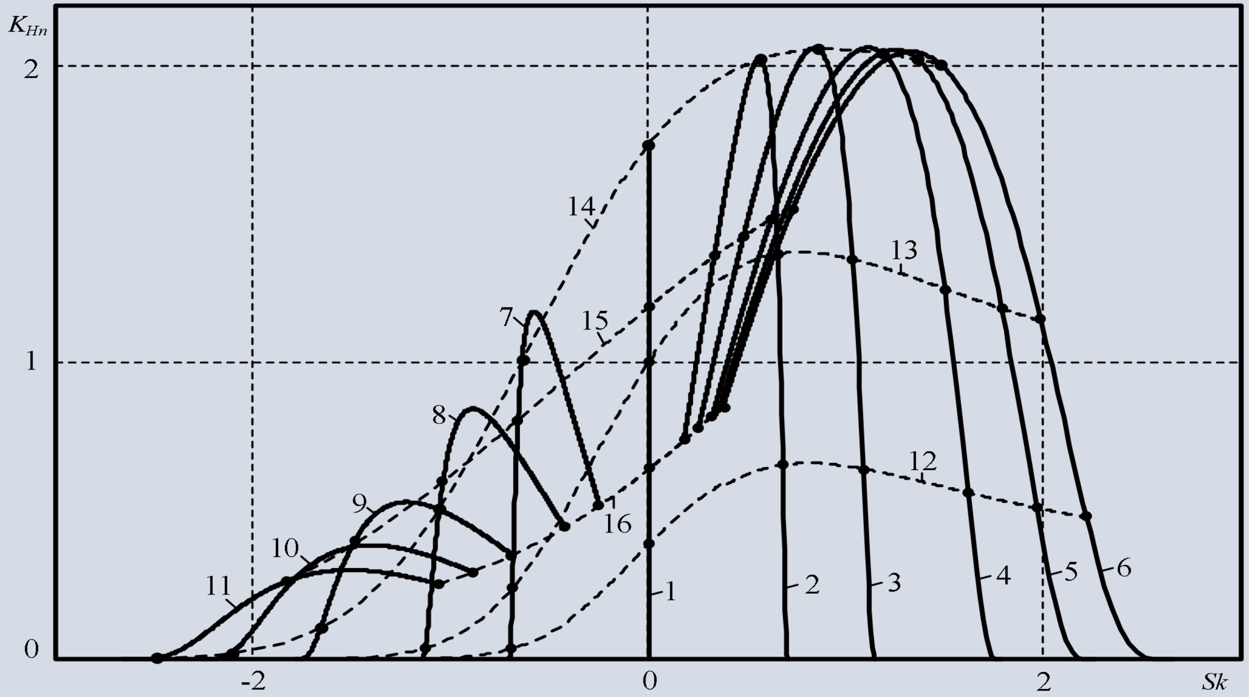
If the *v* second shape parameter is larger than the *u* first parameter by 2, 3, 5 and 15 times, there are the graphs with numbers of 2, 3, 4 and 5 respectively. In the same place, numbers of 6, 7, 8 and 9 is graphs of the change in the entropy coefficient from the *v* second shape parameter provided that the *u* first shape parameter is also 2, 3, 5 and 15 times greater than the *v* second parameter.

Solution method

This is a well discernibility of models for realizing particle size distributions.



- Mapping of non-shifted beta distribution in the space of shape measures of the entropy coefficient and skewness



Where the numbers of 2, 3, 4, 5 and 6 are the trajectories of possible positions of the beta distributions with positive skewnes that the ratio of the v second shape parameter to the u first is equal to 2, 3, 5, 7, and 9, respectively. Numbers of 7, 8, 9, 10 and 11 are the position trajectories of the beta distribution with negative skewnes that the ratio of the u first shape parameter to the v second is equal to 2, 3, 5, 7, and 9, respectively. Dashed illustrate the realization positions of the beta distributions that the first shape parameter of u remains unchanged. There, the numbers 12, 13, 14, 15 and 16 correspond to the shape parameter that it is 0.25, 0.4, 1, 5 and 20, respectively.

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Conclusions

Combined use of parametric and informational measures enhances shape identification capabilities based on beta distribution models.

The positions of realizations of beta distributions with equal values of the shape parameter u are well distinguishable both for small parameters in the region of parameter values less than one, and in the region greater than one.

- By using the example of beta distributions mapping in the space of asymmetry and the entropy coefficient, it is shown that the synergistic combination of probabilistic and informational measures of the shape allows expanding the possibilities of estimating the shape parameters beta distributions.
- Two methods to display the positions of realizations of beta distributions is proposed. There is trajectories on a constant ratio of shape and realizations position curve on equal values of one parameter.
- In particular, the features of the choice of beta distributions with negative skewness are discussed

Contacts

Vitaly Polosin
Medicin Cybernetics and Computer Science Department
Penza State Universiti
E-mail: polosin-vitalij@yandex.ru