«Metrological Support of Innovative Technologies»
ICMSIT-2020

«Marginal reserve: determination, calculation and estimates»

G S Sadykhov, S S Kudryavtseva
Problem statement

- The residual operating time amounts to
  \[ \zeta(\tau) = \zeta - \tau, \]  
  where \( \zeta > \tau \) will be called residual life of the item.

The mean residual life is
  \[ R(\tau) = E\left( \frac{\zeta(\tau)}{\zeta > \tau} \right). \]  

From the definition (2) the indicator is seen as a numerical characteristic of residual operating time in the time interval which makes it difficult to apply it in practice. Therefore, another indicator "mean residual life over time during the duration is used, which is determined by the formula
  \[ R_t(\tau) = E\left( \zeta_t(\tau) \right), \]
  where
  \[ \zeta_t(\tau) = \begin{cases} \zeta - \tau, & \text{if } \zeta \in (\tau, \tau + t); \\ t, & \text{if } \zeta \geq \tau + t. \end{cases} \]

is continuous-discrete and censored above as a random variable.
Solution methods

• Let us determine the marginal reserve of the item, using the following formula:

\[ \Delta_t(\tau) = R(\tau) - R_t(\tau). \]  \hspace{1cm} (3)

• **Theorem 1.** The following formula is valid:

\[ \Delta_t(\tau) = P_t(\tau)R(\tau+t), \]

where

\[ P_t(\tau) = \frac{P(\tau+t)}{P(\tau)} \]

is conditional reliability function of the item in the time interval \((\tau, \tau+t)\), \(P(\cdot)\) is reliability function of the item during the time given in brackets.

**Theorem 2.** The marginal reserve (3) as a function of time \(t\) decreases monotonically taking values from \(R(\tau)\) to zero, while remaining convex.
Solution methods

Theorem 3. For items with a monotonically increasing failure rate function equal to
\[ \lambda(t) = -P'(t)/P(\tau), \]
the indicator as \( \Delta_t(\tau) \) a function of time \( \tau \) decreases monotonically.

Theorem 4. For items with a monotonically decreasing failure rate function (4), the indicator \( \Delta_t(\tau) \) as a function of time \( \tau \) increases monotonically.

For steady-state values of the item's failure rate, the following theorem holds true.

Theorem 5. Let the failure rate of the item satisfy the condition
\[ \lim_{\tau \to \infty} \lambda(\tau) = z. \]

Then the following limit relation is valid:
\[ \lim_{\tau \to \infty} \Delta_t(\tau) = \frac{1}{z} \exp(-zt). \]
Conclusions

Results, implementation

- Thus, the marginal reserve for nonrestorable items is determined.
- Calculation formulas,
- estimates,
- limit values

have been proved for the determined indicator.
Contacts

G S Sadykhov

Department of Computational Mathematics and Mathematical Physics,
Bauman Moscow State Technical University
E-mail: gsadykhov@gmail.com